

Heavy Exotic Molecules with Charm and Bottom

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We revisit the formation of pion-mediated heavy-light exotic molecules with both charm and bottom and their chiral partners under the general strictures of both heavy-quark and chiral symmetry. The chiral exotic partners with good parity formed using the $(0^+, 1^+)$ multiplet are about twice more bound than their primary exotic partners formed using the $(0^-, 1^-)$ multiplet. The chiral couplings across the multiplets $(0^\pm, 1^\pm)$ cause the chiral exotic partners to unbind, and the primary exotic molecules to be about twice more bound, for $J \leq 1$. Our multi-channel coupling results show that only the charm isosinglet exotic molecules with $J^{PC} = 1^{++}$ binds, which we identify as the reported neutral $X(3872)$. Also, the bottom isotriplet exotic with $J^{PC} = 1^{+-}$ binds, which we identify as a mixture of the reported charged exotics $Z_b^+(10610)$ and $Z_b^+(10650)$. The bound isosinglet with $J^{PC} = 1^{++}$ is suggested as a possible neutral $X_b(10532)$ not yet reported.

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I. INTRODUCTION

A decade ago, both the BaBar collaboration [1] and the CLEOII collaboration [2] have reported narrow peaks in the $D_s^+\pi^0$ (2317 MeV) and the $D_s^{*+}\pi^0$ (2460 GeV) channels as expected from general chiral symmetry arguments [3, 4]. In QCD the light quark sector (u, d, s) is dominated by the spontaneous breaking of chiral symmetry, while the heavy quark sector (c, b, t) is characterized by heavy-quark symmetry [5]. The combination of both symmetries led to the conclusion that the heavy-light doublet $(0^-, 1^-) = (D, D^*)$ has a chiral partner $(0^+, 1^+) = (\bar{D}, \bar{D}^*)$ that is about one constituent mass heavier [3, 4].

Recently, the Belle collaboration [6] and the BESIII collaboration [7] have reported the observations of multi-quark exotics. A major provider for these exotics is $\Upsilon(10860)$ and its ideal location near the thresholds for $B\bar{B}^*\pi$ (10744) and $B^*\bar{B}^*\pi$ (10790) decays. The smallness of the available phase space in the hadronic decay of $\Upsilon(10860)$ calls for a compound with a long life-time, perhaps in a molecular configuration with heavy meson constituents. Several heavy exotic molecules with quantum numbers uncommensurate with the excited states of charmonia and bottomia have been reported, such as the neutral $X(3872)$ and the charged $Z_c(3900)^\pm$ and $Z_b(10610)^\pm$. More of these exotics are expected to be unravelled by the DO collaboration at Fermilab [8], and the LHCb collaboration at Cern [9].

Theoretical arguments have predicted the occurrence of some of these exotics as molecular bound states mediated by one-pion exchange much like deuterons or deuterons [10, 11]. A number of molecular estimates regarding the occurrence of doubly heavy exotic mesons with both charm and bottom content were suggested by

many [11–16]. Non-molecular heavy exotics were also discussed using constituent quark models [17], heavy solitonic baryons [18, 19], instantons [20] and QCD sum rules [21]. The molecular mechanism favors the formation of shallow bound states near threshold, while the non-molecular mechanism suggests the existence of deeply bound states. The currently reported exotics by the various experimental collaborations are in support of the molecular configurations.

The purpose of this paper is to revisit the formation of heavy-light molecules under the general strictures of chiral and heavy quark symmetry, including the mixing between the heavy doublets and their chiral partners which was partially considered in [11–15]. In leading order, chiral symmetry fixes the intra- and cross-multiplet couplings. In particular, bound molecules $\bar{D}D$ with charm and $\bar{B}B$ with bottom may form through channel mixing, despite the absence of a direct pion coupling by parity. The P-wave inter-multiplet mixing in the $(0^-, 1^-)$ is enhanced by the almost degeneracy of the constituents by heavy-quark symmetry, while the S-wave cross-multiplet mixing in the $(0^\pm, 1^\pm)$ is still substantial due to the closeness of the constituents by chiral symmetry. The latter prevents the formation of dual chiral molecules such as $\bar{D}\bar{D}$ with charm and $\bar{B}\bar{B}$ with bottom, as we will show. Throughout, the coupling to the low-lying resonances in the continuum with more model assumptions will be ignored for simplicity. Also interactions mediated by shorter range massive vectors and axials will be mostly cutoff through the use of a core cutoff in the pion mediated potential of 1 GeV. Only the channels with total angular momenta $J \leq 1$ will be discussed.

The organization of the paper is as follows: In section 2 we briefly derive the essential construct for doubly charmed exotic molecules using the strictures of chiral and heavy quark symmetries and explicit the coupled channel problem for the lowest bound states. We also show how the same coupled channel problem carries to the chiral partners. In section 3, we extend our analysis to the doubly bottom exotic molecules and their chiral

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partners. Our conclusions are given in section 4.

II. CHARMED EXOTICS MOLECULES

A. $(0^-, 1^-)$ multiplet

The low energy effective action of heavy-light mesons interacting with pions is constrained by both chiral and heavy quark symmetry. In short, the leading part of the heavy-light Lagrangian for the charmed multiplet $(0^-, 1^-)$ with pions reads [3, 5]

$$\begin{aligned} \mathcal{L} \approx & +2i \left(\bar{D} \partial_0 D + \vec{\bar{D}} \cdot \partial_0 \vec{D} \right) \\ & - \Delta m_D \bar{D} D - \Delta m_{\vec{D}} \vec{\bar{D}} \vec{D} \\ & + i \frac{g_H}{f_\pi} \text{Tr} \partial_i \pi \left(D_i D^\dagger - D D_i^\dagger + \epsilon_{ijk} D_k D_j^\dagger \right) \end{aligned} \quad (1)$$

with $\Delta m_i = m_i - m_C$ of the order of a quark constituent mass. The molecular exotics of the type $D\bar{D}^*$ and alike, follows from (1) through one-pion exchange. The non-relativistic character of the molecules yield naturally to a Hamiltonian description.

For all available 2-body channels, the pertinent matrix entries for the interaction are readily found in the form

$$\begin{aligned} \langle \vec{v}_3 \vec{v}_4^* | V | \vec{v}_1 \vec{v}_2^* \rangle &= -C (\vec{v}_3 \times \vec{v}_1) \cdot \nabla (\vec{v}_2 \times \vec{v}_4) \cdot \nabla V(r) \\ \langle 0\bar{0} | V | \vec{v}_1 \vec{v}_2^* \rangle &= C \vec{v}_1 \cdot \nabla \vec{v}_2 \cdot \nabla V(r) \\ \langle \vec{v}_2 \bar{0} | V | 0\vec{v}_1^* \rangle &= -C \vec{v}_1 \cdot \nabla \vec{v}_2 \cdot \nabla V(r) \\ \langle 0\vec{v}_3^* | V | \vec{v}_1 \vec{v}_2^* \rangle &= -C \vec{v}_1 \cdot \nabla (\vec{v}_2 \times \vec{v}_3) \cdot \nabla V(r) \\ \langle \bar{0}\vec{v}_3 | V | \vec{v}_1 \vec{v}_2^* \rangle &= C \vec{v}_2 \cdot \nabla (\vec{v}_3 \times \vec{v}_1) \cdot \nabla V(r) \end{aligned} \quad (2)$$

with the isospin factor

$$C = \vec{I}_1 \cdot \vec{I}_2 = \left(\frac{1}{4} \Big|_{I=1}, -\frac{3}{4} \Big|_{I=0} \right) \quad (3)$$

The spin polarizations of D^* and its conjugate \bar{D}^* are referred to as \vec{v} and \vec{v}^* respectively. Here $V(r)$ is the regulated one-pion exchange using the standard monopole form factor by analogy with the pion-nucleon form factor [22]. Denoting by $D_{0\bar{0}}(\vec{r})$ the wave function of the molecular scalar, by $\bar{Y}_{0\bar{i}}(\vec{r})$ and $Y_{i\bar{0}}(\vec{r})$ the wavefunctions of the molecular vectors, and by $T_{i\bar{j}}(\vec{r})$ the wavefunction of the molecular tensors, we can rewrite (2) as

$$\begin{aligned} (VT)_{k\bar{l}} &= C \epsilon_{kim} \epsilon_{l\bar{j}n} \partial_{mn} V T_{i\bar{j}} \\ (VT)_{0\bar{0}} &= C \partial_{i\bar{j}} V T_{i\bar{j}} \\ (V\bar{Y})_{k\bar{0}} &= -C \partial_k \partial_{\bar{j}} V(r) \bar{Y}_{0\bar{j}} \\ (VT)_{0\bar{k}} &= C \epsilon_{k\bar{l}j} \partial_i \partial_j V(r) T_{i\bar{l}} \\ (VT)_{\bar{0}k} &= C \epsilon_{klj} \partial_i \partial_j V(r) T_{i\bar{l}} \end{aligned} \quad (4)$$

The explicit reduction of the molecular wavefunctions will be detailed below, for all channels with $J \leq 1$.

The one-pion mediated interaction is defined with a core cutoff $\Lambda \gg m_\pi$ [11, 22]

$$\begin{aligned} V(r) &= \left(\frac{g_H}{f_\pi} \right)^2 \\ & \frac{1}{4\pi} \left(\frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda r}}{r} - (\Lambda^2 - m_\pi^2) \frac{e^{-\Lambda r}}{2\Lambda} \right) \end{aligned} \quad (5)$$

Once inserted in (4) it contributes a scalar and a tensor through

$$\partial_i \partial_j V(\vec{r}) = \delta_{ij} V_1(r) + r_i r_j V_2(r) \quad (6)$$

which are shown in Fig. 1 for $g_H = 0.6$ [3, 4] and $\Lambda = 1$ GeV in units of Λ . The strength of the regulated one-pion exchange potential increases with increasing cutoff Λ . The dependence of the results on the choice of core cutoff Λ is the major uncertainty of the molecular analysis to follow. The tensor contribution in (6) is at the origin of the notorious D-wave admixing in the deuteron state [22], and is distinctly different from the gluonic based exchanges in heavy quarkonia [17].

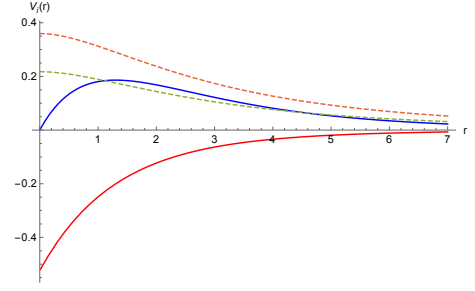


FIG. 1: Typical $V_1(r)$ (lower-full-red) and $V_2(r)$ (upper-full-blue) pion induced potentials compared to $\Delta_1^2 V(r)$ (lower-green-dashed) and $\Delta_2^2 V(r)$ (upper-ored-dashed) to be defined below.

B. $(0^+, 1^+)$ chiral partners and their mixing

The leading part of the heavy-light chiral doublers Lagrangian for the charmed $(0^+, 1^+)$ multiplet with pions reads [3]

$$\begin{aligned} \tilde{\mathcal{L}} \approx & +2i \left(\vec{\bar{D}} \partial_0 \vec{D} + \vec{\bar{D}} \cdot \partial_0 \vec{D} \right) \\ & - \Delta m_{\vec{D}} \vec{\bar{D}} \vec{D} - \Delta m_{\vec{\bar{D}}} \vec{\bar{D}} \vec{D} \\ & + i \frac{g_H}{f_\pi} \text{Tr} \partial_i \pi \left(i(\vec{D}_i \vec{D}^\dagger + \vec{D} \vec{D}_i^\dagger) + \epsilon_{ijk} \vec{D}_k \vec{D}_j^\dagger \right) \end{aligned} \quad (7)$$

with again $\Delta m_i = m_i - m_C$ of the order of a quark constituent mass. The $(0^+, 1^+)$ multiplet mixes with the $(0^-, 1^-)$ by chiral symmetry. The leading part of the interaction in the chirally mixed parity channels reads [3, 4]

$$\delta\mathcal{L} = \frac{g_{HG}}{f_\pi} \text{Tr} \partial_0 \pi \left(\tilde{D}_i^\dagger D_i - i \tilde{D}^\dagger D + \text{c.c.} \right) \quad (8)$$

C. $J=0$ channels

To analyze the coupled molecular ground states, we present the analysis for the $J = 0$ coupled channels. We first discuss the mixing in the $(0^-, 1^-)$ multiplet, followed by the mixing in the $(0^+, 1^+)$ chiral mirror multiplet, and finally the cross mixing between the $(0^\pm, 1^\pm)$ multiplets. The pertinent 0^{PC} channels with their spectroscopic $^S L_J$ assignments are

$$\begin{aligned} 0^{++} : & \quad T_{ij}^{00}(^1 S_0), T_{ij}^{22}(^5 D_0), D_{00}^0(^1 S'_0) \\ 0^{-\pm} : & \quad \bar{Y}_{0i}^1(^3 P_0), Y_{i0}^1(^3 P'_0), T_{ij}^{11}(^3 P''_0) \end{aligned} \quad (9)$$

We have added the primes to track the different contributions in the numerical results below. Here, $T_{ij}^{SL, JM}$ refers to the tensor spherical harmonics with spin S , orbital angular momentum L , and total angular momentum J and projection $J_z = M$. As all $JM = 00$, we have omitted them in (9) for convenience. Also $Y_{0i}^L \equiv Y_i^{L, JM}$ refers to the vector spherical harmonics with orbital angular momentum L , total angular momentum J with $J_z = M$. The explicit form of the properly normalized tensor and vector spherical harmonics in this case, are readily obtained as

$$\begin{aligned} T_{ij}^{00} &= \frac{\delta_{ij}}{\sqrt{3}}, & T_{ij}^{22} &= \sqrt{\frac{3}{2}} \left(\hat{r}_i \hat{r}_j - \frac{\delta_{ij}}{3} \right) \\ \bar{Y}_{0i}^1 &= \hat{r}_i, & Y_{i0}^1 &= \hat{r}_i, & T_{ij}^{11} &= \frac{\epsilon_{ijk} \hat{r}_k}{\sqrt{2}} \end{aligned} \quad (10)$$

Here, we note that $T_{ij}^{00}, T_{ij}^{11}, T_{ij}^{22}$ carries explicitly charge conjugation $C = +$. However, \bar{Y}_{0k}^1, Y_{k0}^1 carry $C = \pm$. It is straightforward to project onto states of good C and rewrite the interactions to follow in this basis, but for $J = 0$ it is not needed, as only the $C = +$ combination is seen not to vanish. It will not be the case for $J = 1$ as we will discuss below. The even-parity channels T^0, T^2, D^0 mix, and the odd-parity channels T^1, Y^1, \bar{Y}^1 mix.

D. Interaction in $(0^-, 1^-)$ multiplet: $J = 0$

The mixing part of the interaction in the $J^{PC} = 0^{++}$ channel is

$$V^{0++} = C \begin{pmatrix} 2V_1 + \frac{2V_2}{3} & -\frac{\sqrt{2}V_2}{3} & \sqrt{3}V_1 + \frac{V_2}{\sqrt{3}} \\ -\frac{\sqrt{2}V_2}{3} & -V_1 + \frac{V_2}{3} & \sqrt{\frac{2}{3}}V_2 \\ \sqrt{3}V_1 + \frac{V_2}{\sqrt{3}} & \sqrt{\frac{2}{3}}V_2 & 0 \end{pmatrix} \quad (11)$$

while in the $J^{PC} = 0^{-\pm}$ channel it is

$$V^{0-\pm} = C \begin{pmatrix} 0 & -V_1 - V_2 & -\sqrt{2}V_1 \\ -V_1 - V_2 & 0 & \sqrt{2}V_1 \\ -\sqrt{2}V_1 & \sqrt{2}V_1 & V_1 + V_2 \end{pmatrix} \quad (12)$$

We note that while the one-pion mediated $D\bar{D} \rightarrow D\bar{D}$ interaction in (11) vanishes by parity, the cross interactions $D\bar{D} \rightarrow D^*\bar{D}^*$ and $D^*\bar{D} \rightarrow D\bar{D}$ do not. As a result bound states through mixing $D\bar{D} \rightarrow D^*\bar{D}^* \rightarrow D\bar{D}$ could and will form in the same order. The corresponding mass shifts and kinetic terms are

$$K^{0++} = \begin{pmatrix} 4m_1 - \frac{\nabla_r^2}{2m_1} & 0 & 0 \\ 0 & 4m_1 - \frac{\nabla_r^2}{2m_1} + \frac{3}{m_1 r^2} & 0 \\ 0 & 0 & 4m_2 - \frac{\nabla_r^2}{2m_2} \end{pmatrix} \quad (13)$$

and

$$K^{0-\pm} = \begin{pmatrix} m_{12} - \frac{\nabla_r^2}{2m_3} + \frac{1}{m_3 r^2} & 0 & 0 \\ 0 & 2m_{12} - \frac{\nabla_r^2}{2m_3} + \frac{1}{m_3 r^2} & 0 \\ 0 & 0 & 4m_1 - \frac{\nabla_r^2}{2m_1} + \frac{1}{m_1 r^2} \end{pmatrix} \quad (14)$$

with $\nabla_r^2 \phi = \frac{1}{r} \frac{d^2}{dr^2} (r\phi)$, $m_{12} = m_1 + m_2$ and the corresponding reduced masses are

$$\left(m_1 \equiv \frac{m_{\bar{D}}}{2}, \quad m_2 \equiv \frac{m_D}{2}, \quad m_3 \equiv \frac{m_D m_{\bar{D}}}{m_D + m_{\bar{D}}} \right) \quad (15)$$

The empirical masses are $m_{D^\pm} = 1.870$ GeV, $m_{D^0} = 1.865$ GeV and $m_{D^{*\pm}} = 2.010$ GeV, $m_{D^{*0}} = 2.007$ GeV. Below, we will use the averages over the isotriplets for $m_{1,2,3}$. Specifically, $m_1 = 1.005$ GeV, $m_2 = 0.934$ GeV and $m_3 = 0.968$ GeV. Here $m_\pi = 137$ MeV and $f_\pi = 93$ MeV, with $g_H = 0.6$ [3, 4].

E. Interaction in $(0^+, 1^+)$ multiplet: $J = 0$

For the $(0^+, 1^+)$ multiplet, the classification of all the states remains the same. The relation between the matrix elements in the $(0^-, 1^-)$ sector and the $(0^+, 1^+)$ sector (primed below) can be made explicit if we note the relations

$$\begin{aligned} |0\rangle' &= -i |0\rangle \\ |\bar{0}\rangle' &= +i |0\rangle \end{aligned} \quad (16)$$

With this in mind, the matrix elements between the different tensor projections are related as follows

$$\begin{aligned} \langle T|H|T\rangle' &= +\langle T|H|T\rangle \\ \langle \bar{Y}|H|T\rangle' &= +i\langle \bar{Y}|H|T\rangle \\ \langle Y|H|T\rangle' &= -i\langle Y|H|T\rangle \\ \langle \bar{Y}|H|Y\rangle' &= -\langle \bar{Y}|H|Y\rangle \end{aligned} \quad (17)$$

As a result we have $\tilde{V}^{0++} = V^{0++}$ and

$$\tilde{V}^{0-\pm} = C \begin{pmatrix} 0 & V_1 + V_2 & -i\sqrt{2}V_1 \\ V_1 + V_2 & 0 & -i\sqrt{2}V_1 \\ i\sqrt{2}V_1 & i\sqrt{2}V_1 & V_1 + V_2 \end{pmatrix} \quad (18)$$

The kinetic contributions \tilde{K}^{0++} and \tilde{K}^{0+-} follows from (13-14) with the appropriate substitution for the reduced masses. We will use the empirical masses for the reported chargeless doublet (D_0^*, D_1^0) with $m_{\bar{D}} = 2.400$ GeV and $m_{\bar{D}^*} = 2.420$ GeV, which translate to $\tilde{m}_1 = 1.210$ GeV, $\tilde{m}_2 = 1.200$ GeV and $\tilde{m}_3 = 1.205$ GeV. Here $\tilde{g}_H = 0.6$ follows from heavy quark symmetry.

F. Interaction across $(0^\pm, 1^\pm)$ multiplets: $J = 0$

The mixed coupling between the $(0^-, 1^-)$ and $(0^+, 1^+)$ induces a scalar interaction typically of the form $\delta V(r) \approx \Delta m^2 V(r)$ with $\Delta m/m_1 \approx 0.4/1.2 = 1/3$. In the relevant range shown in Fig. 1, it is about the same as $V_1(r)$ and will be retained. The corresponding one-pion mediated potential in the 0^{++} is

$$W^{0++} = \left(\frac{g_{GH}}{g_H}\right)^2 CV \begin{pmatrix} -\Delta_1^2 & 0 & 0 \\ 0 & -\Delta_1^2 & 0 \\ 0 & 0 & -\Delta_2^2 \end{pmatrix} \quad (19)$$

and in the $0^{-\pm}$ channel, is

$$W^{0-\pm} = \left(\frac{g_{GH}}{g_H}\right)^2 CV \begin{pmatrix} i\Delta_1\Delta_2 & 0 & 0 \\ 0 & -i\Delta_1\Delta_2 & 0 \\ 0 & 0 & -\Delta_1^2 \end{pmatrix} \quad (20)$$

Here the empirical mass splittings are

$$\begin{aligned} \Delta_1 &= (m_{\bar{D}^*} - m_{D^*}) \approx 410 \text{ MeV} \\ \Delta_2 &= (m_{\bar{D}} - m_D) \approx 530 \text{ MeV} \end{aligned} \quad (21)$$

The stationary coupled channel problem for the ground states in $J^{PC} = 0^{++}$ and $J^{PC} = 0^{-+}$, follows from the

6×6 eigenvalue problem $(\mathbf{H} = \mathbf{K} + \mathbf{V} + \mathbf{W}) \Phi_i = E \Phi_i$ with now $\Phi_i \equiv r \phi_i$. To proceed further, we need to solve the coupled channels problem numerically with

$$\mathbf{H} = \begin{pmatrix} K + V & W^\dagger \\ W & \tilde{K} + \tilde{V} \end{pmatrix} \quad (22)$$

in each sector.

G. $J = 1$ channels

The pertinent projections onto the higher J^{PC} channels of the molecular wavefunctions in (4) require the use of both vector and higher tensor spherical harmonics [23, 24]. For $J = 1$, we will use the explicit forms quoted in [24] with the $^S L_J$ assignment completely specified. For the $(1^\mp, 0^\mp)$ multiplets, there are 4 different 1^{PC} sectors

$$\begin{aligned} 1^{++} : & \quad T_{ij}^{2,2}({}^5D_1), Y_i^{0+}({}^3S_1), Y_i^{2+}({}^3D_1) \\ 1^{--} : & \quad T_{ij}^{0,1}({}^1P_1), T_{ij}^{2,1}({}^5P_1), T_{ij}^{2,3}({}^5F_1), \\ & \quad Y_i^{1-}({}^3P_1), D^1({}^1P'_1) \\ 1^{+-} : & \quad T_{ij}^{1,0}({}^3S_1), T_{ij}^{1,2}({}^3D_1), Y_i^{0-}({}^3S'_1), Y_i^{2-}({}^3D'_1) \\ 1^{-+} : & \quad T_{ij}^{1,1}({}^3P_1), Y_i^{1+}({}^3P'_1) \end{aligned} \quad (23)$$

with the JM labels omitted for convenience. The normalized tensor harmonics are [24]

$$\begin{aligned} T_{ij}^{01,1m} &= \frac{\delta_{ij}}{\sqrt{3}} Y_{1m} \\ T_{ij}^{21,1m} &= \sqrt{\frac{3}{5}} \left(\frac{\delta_{ij}}{3} - \hat{r}_i \hat{r}_j \right) Y_{1m} \\ &\quad - \sqrt{\frac{3}{10}} (r_i \nabla_j + r_j \nabla_i) Y_{1m} \\ T_{ij}^{23,1m} &= 3 \sqrt{\frac{1}{10}} \left(\frac{\delta_{ij}}{3} - \hat{r}_i \hat{r}_j \right) Y_{1m} \\ &\quad - \sqrt{\frac{1}{5}} (r_i \nabla_j + r_j \nabla_i) Y_{1m} \\ T_{ij}^{22,1m} &= \frac{1}{2} (r_i L_j + r_j L_i) Y_{1m} \\ Y_i^{0,1m} &= \frac{1}{\sqrt{3}} (\sqrt{2} r \nabla_i Y_{1m} + \hat{r}_i Y_{1m}) \\ Y_i^{2,1m} &= \frac{1}{\sqrt{3}} (r \nabla_i Y_{1m} - \sqrt{2} \hat{r}_i Y_{1m}) \\ Y_i^{1,1m} &= \frac{1}{\sqrt{2}} i L_i Y_{1m} \end{aligned} \quad (24)$$

The DD^* channels with definite charge conjugation $C = \pm$ are explicitly

$$\begin{aligned}
Y_i^{0\pm} &= \frac{1}{\sqrt{2}}(\bar{Y}_{0i}^0 \pm Y_{i0}^0) \\
Y_i^{2\pm} &= \frac{1}{\sqrt{2}}(\bar{Y}_{0i}^2 \pm Y_{i0}^2) \\
Y_i^{1\pm} &= \frac{1}{\sqrt{2}}(\bar{Y}_{0i}^1 \mp Y_{i0}^1)
\end{aligned} \tag{25}$$

We note that

$$T_{ij}^{1L, JM} = \frac{\epsilon_{ij\bar{k}}}{\sqrt{2}} Y_k^{L, JM} \tag{26}$$

H. Interaction in $(0^-, 1^-)$ multiplet: $J = 1$

In terms of the previous $J = 1$ channels, the one-pion mediated interaction in the $(0^-, 1^-)$ multiplet in the $J^{PC} = 1^{--}$ channel takes the block form

$$V^{1--} = C \begin{pmatrix} v_0 & v_1 \\ v_1^\dagger & v_2 \end{pmatrix} \tag{27}$$

with the blocks defined as

$$v_0 = \begin{pmatrix} 2V_1 + \frac{2V_2}{3} & \frac{2V_2}{3\sqrt{5}} & -\sqrt{\frac{2}{15}}V_2 \\ \frac{2V_2}{3\sqrt{5}} & -V_1 + \frac{2V_2}{15} & -\frac{\sqrt{6}}{15}V_2 \\ -\sqrt{\frac{2}{15}}V_2 & -\frac{\sqrt{6}}{15}V_2 & -V_1 + \frac{V_2}{15} \end{pmatrix} \tag{28}$$

$$v_1 = \begin{pmatrix} 0 & \sqrt{3}V_1 + \frac{V_2}{\sqrt{3}} \\ \frac{\sqrt{3}V_2}{\sqrt{5}} & -\frac{2V_2}{\sqrt{15}} \\ \frac{\sqrt{2}V_2}{\sqrt{5}} & \sqrt{\frac{2}{5}}V_2 \end{pmatrix} \tag{29}$$

$$v_2 = \begin{pmatrix} -V_1 & 0 \\ 0 & 0 \end{pmatrix} \tag{30}$$

Similarly, the one-pion mediated interaction in the $J^{PC} = 1^{+-}$ channel has the following block structure

$$V^{1+-} = C \begin{pmatrix} v_3 & v_4 \\ v_4^\dagger & v_5 \end{pmatrix} \tag{31}$$

with each block defined as

$$v_3 = v_5 = \begin{pmatrix} V_1 + \frac{V_2}{3} & -\frac{\sqrt{2}V_2}{3} \\ -\frac{\sqrt{V_2}}{3} & V_1 + \frac{2V_2}{3} \end{pmatrix} \tag{32}$$

$$v_4 = \begin{pmatrix} -2V_1 - \frac{2V_2}{3} & -\frac{\sqrt{2}V_2}{3} \\ -\frac{\sqrt{2}V_2}{3} & -2V_1 - \frac{V_2}{3} \end{pmatrix} \tag{33}$$

In the remaining $J^{PC} = 1^{-+}$ and $J^{PC} = 1^{++}$ the one-pion mediated interactions are respectively given by

$$V^{1-+} = C \begin{pmatrix} V_1 & -2V_1 - V_2 \\ -2V_1 - V_2 & V_1 \end{pmatrix} \tag{34}$$

and

$$V^{1++} = C \begin{pmatrix} -V_1 & i\frac{\sqrt{2}V_2}{\sqrt{3}} & \frac{iV_2}{\sqrt{3}} \\ -i\frac{\sqrt{2}V_2}{\sqrt{3}} & -\frac{3V_1+V_2}{3} & \frac{\sqrt{2}V_2}{3} \\ -\frac{iV_2}{\sqrt{3}} & \frac{\sqrt{2}V_2}{3} & -\frac{3V_1+2V_2}{3} \end{pmatrix} \tag{35}$$

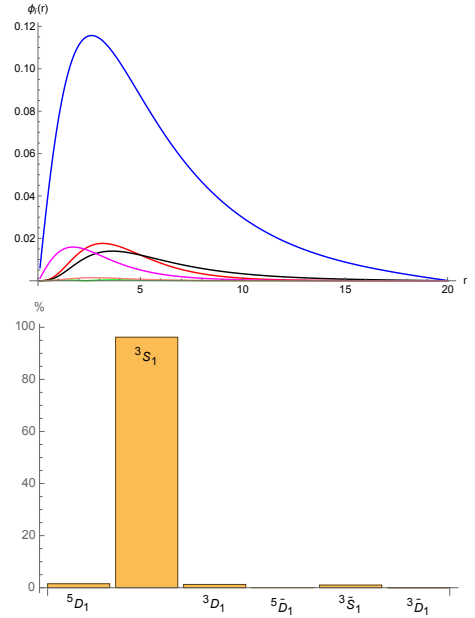


FIG. 2: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{++}$ channel (upper plot), for the lowest bound state for the charm exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 1^{++}$ channel are shown below, with their spectroscopic assignments.

I. Interaction across $(0^\pm, 1^\pm)$ multiplets: $J = 1$

The one-pion mediated interaction within the $(0^+, 1^+)$ multiplet follows the same construct as in the $(0^-, 1^-)$ multiplet using the transfer rules in (16-17). The same interaction across the two chiral multiplets introduces also a diagonal mixing of the form

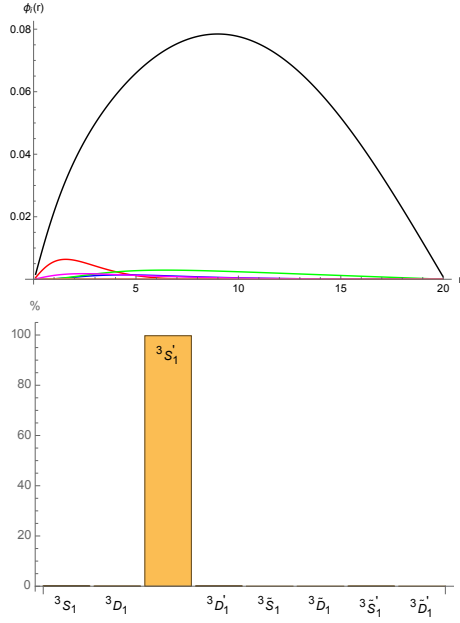


FIG. 3: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{+-}$ channel (upper plot), for the lowest **unbound** state for the charm exotic state with $C = 1/4$ (isotriplet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the **unbound** wavefunction in the $J^{PC} = 1^{+-}$ channel are shown below, with their spectroscopic assignments.

$$\begin{aligned}
 \langle T|V|\tilde{T} \rangle &= -CV \left(\frac{g_{GH}}{g_H} \right)^2 \Delta_1^2 \delta_{T\tilde{T}} \\
 \langle D|V|\tilde{D} \rangle &= -CV \left(\frac{g_{GH}}{g_H} \right)^2 \Delta_2^2 \delta_{D\tilde{D}} \\
 \langle Y|V|\tilde{Y} \rangle &= -iCV \left(\frac{g_{GH}}{g_H} \right)^2 \Delta_1 \Delta_2 \delta_{Y\tilde{Y}} \quad (36)
 \end{aligned}$$

The $\tilde{D}\tilde{D}^*$ states with good charge conjugation follows from (25) through the substitution $\pm \rightarrow \mp$ only on the right hand side. The total Hamiltonian in the $(0^+, 1^+)$ sector to diagonalize is $H = K + V$. Including the chiral multiplet, the total Hamiltonian across the $(0^\pm, 1^\pm)$ sectors to diagonalize is twice larger $\mathbf{H} = \mathbf{K} + \mathbf{V} + \mathbf{W}$.

J. Results for charm exotic molecules

In the upper plot of Fig. 2 we show the typical Φ_i radial components of the bound isosinglet charm wavefunction with energy $E = 3.867$ GeV for a cutoff $\Lambda = 1$ GeV, as a function of the radial distance r also in units of 1 GeV. The chiral cross coupling between the $(0^-, 1^-)$ and $(0^+, 1^+)$ multiplets induces a very small mixing to the molecular wavefunction in the $(0^-, 1^-)$ multiplet as displayed in Fig. 2. In the lower chart of Fig. 2 we show the percentage content of the contributions to the same

wavefunction, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^S \tilde{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet. The mixing results in a stronger binding in this channel which is mostly an isosinglet $^1 S_3$ contribution in the $(1^-, 0^-)$ multiplet with almost no D-wave admixture. This molecular state carries $J^{PC} = 1^{++}$ assignment, and from our $^S L_J$ assignments in (24) it is chiefly an isosinglet $D\tilde{D}^*$ molecule. We identify this state with the reported isosinglet exotic $X(3872)$. The cross chiral mixing causes the dual chiral partners $\tilde{D}\tilde{D}^*$ state to unbind. .

In the upper plot of Fig. 3 we show the typical Φ_i radial components of the **unbound** isotriplet charm wavefunction for a cutoff $\Lambda = 1$ GeV, as a function of the radial distance r also in units of 1 GeV. The multi-channel coupling in the channel with $J^{PC} = 1^{+-}$, shows that the dominant wave is $^3 S_1'$ which is composed of a resonating isosinglet $D\tilde{D}^*(3876)$ compound. The wave shows a weak visible attraction near the origin that is not enough to bind. We conclude that the reported $Z_c(3900)^\pm$ is at best a resonance in the continuum in our analysis. All other channels are unbound for charm exotic molecules with our cutoff of 1 GeV, in both the isosinglet and isotriplet configurations

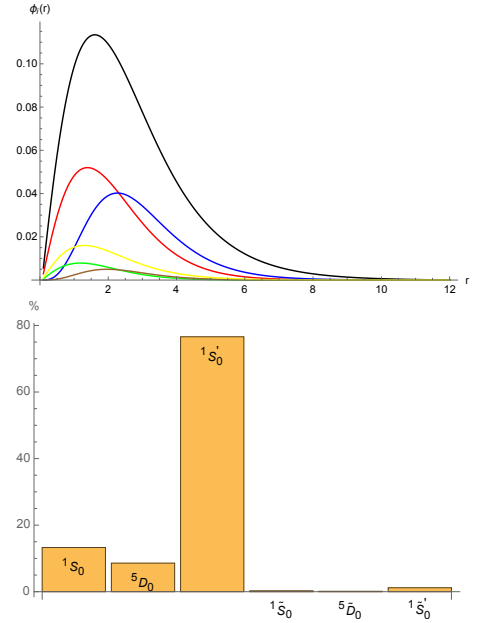


FIG. 4: Typical Φ radial wavefunctions in the mixed $J^{PC} = 0^{++}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 0^{++}$ channel are shown below, with their spectroscopic assignments.

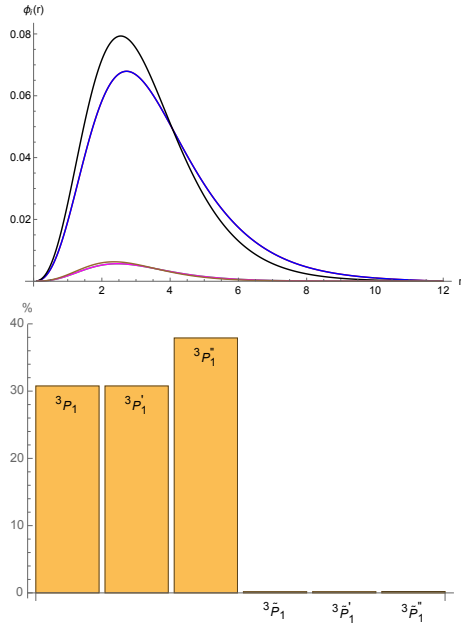


FIG. 5: Typical Φ radial wavefunctions in the mixed $J^{PC} = 0^{-+}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 0^{-+}$ channel are shown below, with their spectroscopic assignments.

III. BOTTOM EXOTIC MOLECULES

Doubly bottom exotic molecules follow the same construction as before, all potentials and interactions remains of the the same form , with now the new mass parameters

$$\left(m_1 \equiv \frac{m_{\tilde{B}}}{2}, \quad m_2 \equiv \frac{m_B}{2}, \quad m_3 \equiv \frac{m_B m_{\tilde{B}}}{m_B + m_{\tilde{B}}} \right) \quad (37)$$

For the $(0^-, 1^-)$ multiplet, we have $m_B = 5.279$ GeV, $m_{\tilde{B}} = 5.325$ GeV and $m_{\tilde{B}} \approx m_{\tilde{B}} = 5.727$ GeV, and therefore $m_1 = 2.662$ GeV, $m_2 = 2.640$ GeV and $m_3 = 2.651$ GeV. For the $(0^+, 1^+)$ multiplet, we have $m_{\tilde{B}} = 5.727$ GeV. Assuming a common splitting $m_{\tilde{B}} - m_B = m_{\tilde{B}} - m_{\tilde{B}} = 46$ MeV, we have $m_{\tilde{B}} = 5.681$ GeV and therefore $\tilde{m}_1 = 2.869$ GeV, $\tilde{m}_2 = 2.840$ GeV and $\tilde{m}_3 = 2.852$ GeV. The results for the chirally mixed states for the bottom exotic states involving the pair multiplet $(0^\pm, 1^\pm)$ can be obtained using similar arguments to those used for charm with the same cutoff choice. Since the one-pion exchange interaction is three times stronger in the isosinglet channel than the isotriplet channel, a multitude of isosinglet bottom exotic states will be revealed, thanks also to the heavier bottom mass and thus smaller kinetic energy in comparison to the charm exotic states.

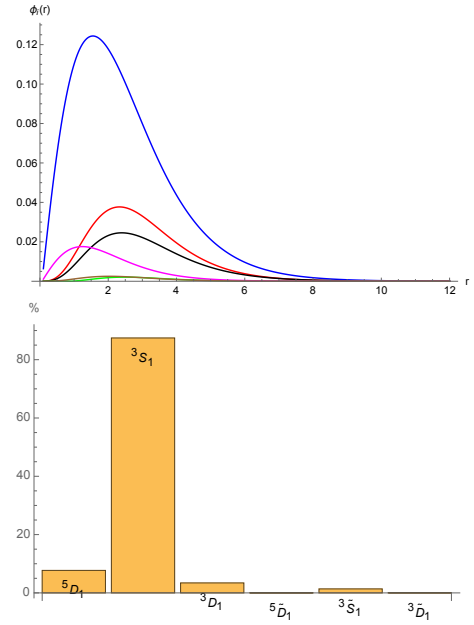


FIG. 6: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{++}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 1^{++}$ channel are shown below, with their spectroscopic assignments.

A. Results for bottom exotic molecules

In Fig. 4 we show the behavior of the typical isosinglet bound state wavefunctions contributing in the $J^{PC} = 0^{++}$ channel with energy $E = 10.509$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^S \tilde{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet. From the assignments given in (9), we see that the 0^{++} mixed bound state is chiefly an isosinglet $B\tilde{B}$ ($^1 S_0$) molecule, with relatively small $B^* \tilde{B}^*$ ($^1 S_0$) and $B^* \tilde{B}^*$ ($^5 D_0$) admixtures.

In Fig. 5 we show the behavior of the typical isosinglet bound state wavefunctions contributing in the $J^{PC} = 0^{-+}$ channel with energy $E = 10.555$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^S \tilde{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet. From the assignments given in (9), we see that the 0^{-+} mixed bound state is a mixed molecule with about equal admixture of $B\tilde{B}^*$ ($^3 P_0$), $B^* \tilde{B}$ ($^3 P'_0$) and $B^* \tilde{B}^*$ ($^3 P''_0$) molecules all from the $(0^-, 1^-)$ multiplet as those from the $(0^+, 1^+)$ are shown to decouple and unbind. The effect of the latters is to cause the formers to bind twice

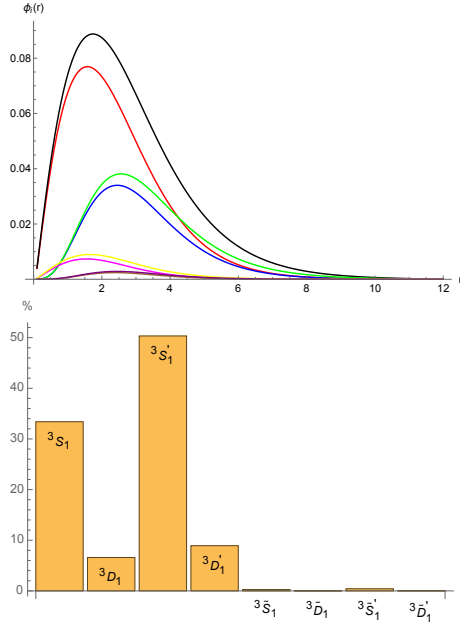


FIG. 7: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{+-}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 1^{+-}$ channel are shown below, with their spectroscopic assignments.

more.

In Fig. 6 we show the behavior of the typical isosinglet bound wavefunctions contributing in the $J^{PC} = 1^{++}$ channel with energy $E = 10.532$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^{\tilde{S}} \tilde{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet. From the assignments given in (24), we see that the 1^{++} mixed bound state is chiefly a $B\bar{B}^* (^3S_1)$, with small $B^*\bar{B}^* (^5D_1)$ and $B\bar{B}^* (^3D_1)$ admixtures. We see again the decoupling of the molecular configurations with $^{\tilde{S}} \tilde{L}_J$ assignments as they are found to unbind, leaving the $^S L_J$ assignments twice more bound as per our calculation. A quick comparison between Fig. 2 and Fig. 6 shows that this neutral bottom molecular state is the mirror analogue of the neutral charm molecular state which we suggest as $X_b(10532)$.

In Fig. 7 we show the behavior of the typical isosinglet bound wavefunctions contributing in the $J^{PC} = 1^{+-}$ channel with energy $E = 10.550$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^{\tilde{S}} \tilde{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet. From the assignments given in (24), we see that the 1^{+-} mixed

bound state are primarily $B^*\bar{B}^* (^3S_1)$ and $B\bar{B}^* (^3S_1')$ molecules, with small $B^*\bar{B}^* (^3D_1)$ and $B\bar{B}^* (^3D_1')$ molecular admixtures. The molecules are mostly from the $(0^-, 1^-)$ multiplet as those from the $(0^+, 1^+)$ are shown to decouple and unbind. Again, the effect of the latters is to cause the formers to bind twice more.

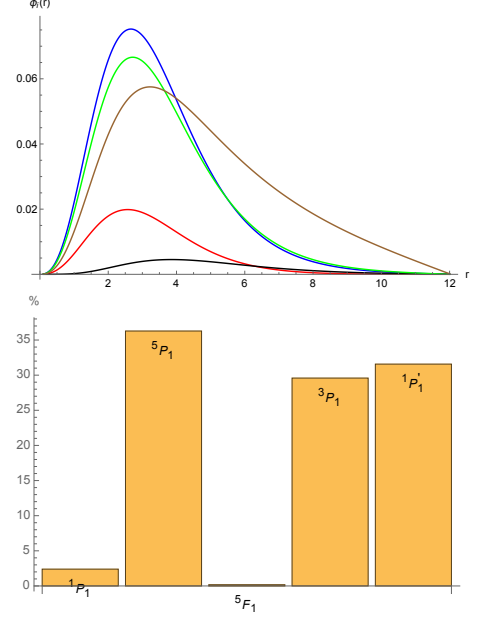


FIG. 8: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{--}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = -3/4$ (isosinglet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 1^{--}$ channel are shown below, with their spectroscopic assignments.

In Fig. 8 we show the behavior of the typical isosinglet bound wavefunctions contributing in the $J^{PC} = 1^{--}$ channel with energy $E = 10.558$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet only. From the assignments given in (24), we see that the 1^{--} isosinglet bound state is mostly P-wave with equal admixture of $B^*\bar{B}^* (^5P_1)$, $B\bar{B}^* (^3P_1)$ and $B\bar{B} (^1P_1')$ molecules. We note the clear repulsion of the P-waves near the origin. The molecules are mostly from the $(0^-, 1^-)$ multiplet as those from the $(0^+, 1^+)$ decouple and unbind. The effect of the latters is to cause the formers to bind twice more. This isosinglet molecular exotic is well below the reported $Y_b(10888)$.

In Fig. 9 we show the behavior of the typical isotriplet bound wavefunctions contributing in the $J^{PC} = 1^{+-}$ channel with energy $E = 10.592$ GeV ($\Lambda = 1$ GeV) (upper-plots). The percentage content of the same wavefunction is displayed as a histogram with the appropriate parity labels in the lower display, with the $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^{\tilde{S}} \tilde{L}_J$

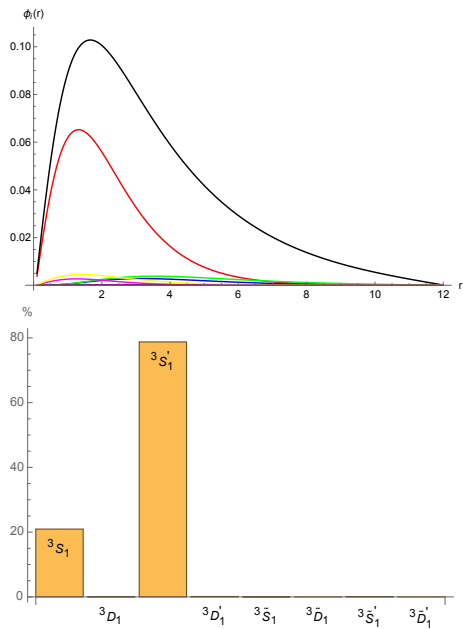


FIG. 9: Typical Φ radial wavefunctions in the mixed $J^{PC} = 1^{+-}$ channel (upper plot), for the lowest bound state for the bottom exotic state with $C = +1/4$ (isotriplet), in units of $\Lambda = 1$ GeV. The corresponding percentages content of the bound wavefunction in the $J^{PC} = 1^{+-}$ channel are shown below, with their spectroscopic assignments.

assignments referring to the $(0^+, 1^+)$ multiplet. From the assignments given in (24), we see that the 1^{+-} mixed isotriplet bound state is mostly an S-state made primarily of $B\bar{B}^*$ ($^3S'_1$) molecules with a small admixture of $B^*\bar{B}^*$ ($^3S'_1$) molecules. The molecules are mostly from the $(0^-, 1^-)$ multiplet as those from the $(0^+, 1^+)$ again decouple and unbind. We identify this exotic molecule as a mixed state of the reported pair of isotriplet exotics $Z_b^+(10610)$ and $Z_b^+(10650)$.

IV. CONCLUSIONS

We have analyzed molecular states of doubly heavy mesons mediated by one-pion exchange for both the chiral partners $(0^\pm, 1^\pm)$ as a coupled channel problem, for all the molecular configurations with $J \leq 1$. Our results show that the binding energy is sensitive to the cutoff used for the one-pion exchange interaction which is substantial in the lowest partial waves. All other parameters are fixed by symmetry and data. Our results complement and extend those presented in [11–16] by taking into account the strictures of chiral and heavy quark symmetry, and by retaining most coupled channels between the $(0^-, 1^-)$ multiplet and its chiral partner $(0^+, 1^+)$. The key aspect of this coupling is to cause the molecules in the $(0^-, 1^-)$ multiplet to bind about twice more, and the molecules in the $(0^+, 1^+)$ multiplet to unbind.

For channel couplings with $J \leq 1$, we have found that only the charm isosinglet exotic molecules with $J^{PC} = 1^{++}$ is strictly bound for a pion-exchange cutoff $\Lambda = 1$ GeV. This state is identified with the reported isosinglet exotic $X(3872)$ which in our case is mostly an isosinglet $D\bar{D}^*$ molecule in the 1S_0 channel with no D-wave admixture. The attraction in the isotriplet channel with $J^{PC} = 1^{+-}$ is too weak to bind the $D\bar{D}^*$ compound, suggesting that the reported isotriplet $Z_C(3900)^\pm$ is at best a near threshold resonance. All other J^{PC} assignments with charm for both the isotriplet and isosinglet are unbound. The noteworthy absence in our analysis of the $Y(4260)$, $Y(4360)$ and $Y(4660)$ may point to the possibility of their constituents made of excited (D_1, D_2) heavy mesons and their chiral partners [3, 25], which we have not considered.

In contrast, and for the same choice of the cutoff, we have identified several isosinglet bottom exotic molecules in the $J^{PC} = 0^{\pm+}, 1^{+\pm}, 1^{--}$ channels which are mostly admixtures of the heavy-light mesons in the $(0^-, 1^-)$ multiplet. We have only found one isotriplet bottom exotic molecule with $J^{PC} = 1^{+-}$ which we have identified with the pair $Z_b^+(10610)$ and $Z_b^+(10650)$, which is a mixed state in our analysis. The isosinglet bottom exotic molecule with $J^{PC} = 1^{++}$ is a potential candidate for $X_b(10532)$, the bottom analogue of the charm exotic $X(3872)$.

Our results show that the cross chiral mixing between the $(0^\pm, 1^\pm)$ multiplets while strong, does not generate new mixed molecules of the type $D\bar{D}^*$ and alike, as suggested in [13]. Rather, it prevents the formation of dual chiral molecules of the type $\bar{D}\bar{D}^*$ and alike, which would be otherwise possible. In the process, it provides for a stronger binding of the low lying molecules in the $(0^-, 1^-)$ multiplet in comparison to the results in [14]. Most noteworthy, is the appearance of a single bound isosinglet $J^{PC} = 1^{++}$ charm exotic molecule in our analysis, with also one single bound isotriplet bottom exotic molecule with $J^{PC} = 1^{+-}$ but several isosinglet bottom exotic molecular states with $J^{PC} = 0^{\pm+}, 1^{+\pm}$. The latter may transmute to broad resonances by mixing to bottomia with similar quantum numbers.

Clearly higher values of $J > 1$ may also be considered using the same construct, but the molecular configurations maybe too large to bind, a point in support of their absence in the currently reported experiments. The recoupling of the current bound state problem to the open channels with charmonia and bottomia is also important to consider, but requires a more extensive analysis of the multi-channel scattering problem. Finally, the extension of the present analysis to D_s and B_s molecules using the heavier eta-exchange [26], as well as exotic baryonic molecules should be of interest in light of the ongoing experimental programs.

V. ACKNOWLEDGEMENTS

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- [1] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **90**, 242001 (2003) [hep-ex/0304021].
 - [2] D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 032002 (2003) Erratum: [Phys. Rev. D **75**, 119908 (2007)] [hep-ex/0305100].
 - [3] M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D **48**, 4370 (1993) [hep-ph/9209272]; M. A. Nowak, M. Rho and I. Zahed, Acta Phys. Polon. B **35**, 2377 (2004) [hep-ph/0307102].
 - [4] W. A. Bardeen and C. T. Hill, Phys. Rev. D **49** (1994) 409 [hep-ph/9304265]; W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D **68**, 054024 (2003) [hep-ph/0305049].
 - [5] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66** (1991) 1130; A. V. Manohar and M. B. Wise, “Heavy quark physics,” Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1 (2000).
 - [6] I. Adachi [Belle Collaboration], arXiv:1105.4583 [hep-ex]; A. Bondar *et al.* [Belle Collaboration], Phys. Rev. Lett. **108**, 122001 (2012) [arXiv:1110.2251 [hep-ex]].
 - [7] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].
 - [8] V. M. Abazov *et al.* [D0 Collaboration], [arXiv:1602.07588 [hep-ex]].
 - [9] R. Aaij *et al.* [LHCb Collaboration], arXiv:1606.07895 [hep-ex]; R. Aaij *et al.* [LHCb Collaboration], arXiv:1606.07898 [hep-ex].
 - [10] M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976) [Pisma Zh. Eksp. Teor. Fiz. **23**, 369 (1976)];
 - [11] N. A. Tornqvist, Phys. Rev. Lett. **67**, 556 (1991); N. A. Tornqvist, Z. Phys. C **61**, 525 (1994) [hep-ph/9310247]; N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004) [hep-ph/0402237].
 - [12] M. Karliner and H. J. Lipkin, arXiv:0802.0649 [hep-ph]; M. Karliner and J. L. Rosner, Phys. Rev. Lett. **115** (2015) no.12, 122001 [arXiv:1506.06386 [hep-ph]]; M. Karliner, Acta Phys. Polon. B **47**, 117 (2016).
 - [13] C. E. Thomas and F. E. Close, Phys. Rev. D **78**, 034007 (2008) [arXiv:0805.3653 [hep-ph]]; F. Close, C. Downum and C. E. Thomas, Phys. Rev. D **81**, 074033 (2010) [arXiv:1001.2553 [hep-ph]].
 - [14] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, Phys. Rev. D **86**, 034019 (2012) [arXiv:1202.0760 [hep-ph]]; S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, arXiv:1209.0144 [hep-ph].
 - [15] I. W. Lee, A. Faessler, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **80**, 094005 (2009) [arXiv:0910.1009 [hep-ph]]; M. Suzuki, Phys. Rev. D **72**, 114013 (2005) [hep-ph/0508258]; J. R. Zhang, M. Zhong and M. Q. Huang, Phys. Lett. B **704**, 312 (2011) [arXiv:1105.5472 [hep-ph]]; D. V. Bugg, Europhys. Lett. **96**, 11002 (2011) [arXiv:1105.5492 [hep-ph]]; J. Nieves and M. P. Valderama, Phys. Rev. D **84**, 056015 (2011) [arXiv:1106.0600 [hep-ph]]; M. Cleven, F. K. Guo, C. Hanhart and U. G. Meissner, Eur. Phys. J. A **47**, 120 (2011) [arXiv:1107.0254 [hep-ph]]; T. Mehen and J. W. Powell, Phys. Rev. D **84**, 114013 (2011) [arXiv:1109.3479 [hep-ph]].
 - [16] E. S. Swanson, Phys. Rept. **429**, 243 (2006) [hep-ph/0601110]; Z. F. Sun, J. He, X. Liu, Z. G. Luo and S. L. Zhu, Phys. Rev. D **84**, 054002 (2011) [arXiv:1106.2968 [hep-ph]].
 - [17] A. V. Manohar and M. B. Wise, Nucl. Phys. B **399**, 17 (1993) [hep-ph/9212236]; N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011) [arXiv:1010.5827 [hep-ph]]; M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008) [arXiv:0711.4556 [hep-ph]]; J. M. Richard, arXiv:1606.08593 [hep-ph].
 - [18] D. O. Riska and N. N. Scoccola, Phys. Lett. B **299**, 338 (1993).
 - [19] M. A. Nowak, I. Zahed and M. Rho, Phys. Lett. B **303**, 130 (1993).
 - [20] S. Chernyshev, M. A. Nowak and I. Zahed, Phys. Rev. D **53**, 5176 (1996) [hep-ph/9510326].
 - [21] M. Nielsen, F. S. Navarra and S. H. Lee, Phys. Rept. **497**, 41 (2010) [arXiv:0911.1958 [hep-ph]].
 - [22] G.E. Brown and A.D. Jackson, The nucleon-nucleon interaction, Ed. North Holland (1976).
 - [23] A. R. Edmonds, “Angular momentum in quantum mechanics,” Ed. Princeton Univ. Press (1974).
 - [24] K. S. Thorne, Rev. Mod. Phys. **52**, 299 (1980).
 - [25] M. A. Nowak and I. Zahed, Phys. Rev. D **48**, 356 (1993).
 - [26] M. Karliner and J. L. Rosner, arXiv:1601.00565 [hep-ph].